

Properties of baryon resonances from a multichannel partial wave analysis

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Abstract. Properties of nucleon and Δ resonances are derived from a multichannel partial wave analysis. The statistical significance of pion and photo-induced inelastic reactions off protons are studied in a multichannel partial-wave analysis.

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1 Introduction

Existence and properties of most N and Δ resonances listed in the Review of Particle Properties [1] were derived from partial wave analyses of πN elastic and charge exchange scattering data [2, 3, 4, 5]. Additional information on their decay modes was obtained from inelastic reactions, from $\pi N \rightarrow N\eta, \Lambda K, \Sigma K$ and from an isobar model study of $\pi N \rightarrow N\pi\pi$; photoproduction experiments provided information on the photo-coupling. The most recent analysis [5] – based on a larger data set and on very precise data from meson factories – found no evidence for the existence of 16 of the 32 N and Δ resonances below 2.2 GeV listed in the Baryon Particle Tables. Obviously, the existing data base was not sufficient to extract a reliable spectrum of N and Δ resonances from pion induced reactions alone.

In the last years, an impressive amount of photo-induced reactions has been studied at ELSA, GRAAL, Jlab, MAMI, and SPring-8, and the situation has changed significantly. High-statistics data are available not only on differential cross sections but also on many polarization observables. In particular, reactions like $\gamma p \rightarrow p\pi^0, n\pi^+, p\eta, p\pi^0\pi^0, p\pi^+\pi^-, p\pi^0\eta, \Lambda K^+, \Sigma^0 K^+,$ and $\Sigma^+ K_s^0$ have been studied, some of them in great detail.

In this paper, we give a brief account of the results of the Bonn-Gatchina (BnGa) multichannel partial wave analysis. Main results have been reported before [6, 7, 8, 9]. We found two classes of solutions, called BnGa2011-01 and BnGa2011-02, which differ in the number and properties of some positive-parity nucleon resonances at masses above 1.9 GeV. The emphasis of the papers [7, 9] was on a discussion of the alternative solutions, on the new resonances found in the analysis, and on their physics interpretation. In [6], amplitudes for pion photoproduction off protons were presented, and in [8], the focus was to explore possible interpretations of a narrow structure in the $N\eta$ mass distribution. The emphasis here is to provide complete information on resonances, their masses and widths, their helicity amplitudes, and their decay properties. The statistical evidence

(in terms of χ^2) is given for each resonance and for their decay modes into $N\gamma, N\pi, N\eta, \Lambda K$ and ΣK . The results are not derived from new fits. Only the error analysis has been improved by storing several acceptable solutions and by calculating (instead of estimating) properties and errors from the distribution of all quantities. Hence the results supersede those of [7, 9].

2 Data used in the partial wave analysis

Tables 1-6 give an updated list of the pion- and photo-induced reactions used in the coupled channel analysis presented here. The data comprise nearly all important reactions including mul-

Table 1. Fit to the real and imaginary part of elastic πN amplitudes and χ^2 contributions for the solution BG2011-02. The elastic scattering data are fitted jointly with a larger number of further data in a coupled channel approach.

$\pi N \rightarrow \pi N$	Wave	N_{data}	w_i	χ^2_i/N_{data}
[5]	S_{11}	112	30	2.11
	S_{31}	112	20	2.19
	P_{11}	112	70	1.70
	P_{31}	104	20	3.74
	P_{13}	112	25	1.39
	P_{33}	120	15	2.77
	D_{13}	108	10	2.21
	D_{33}	108	12	3.08
	D_{15}	104	20	2.29
	F_{15}	88	30	1.87
	F_{35}	62	20	1.64
	F_{37}	72	10	2.76
	F_{17}	82	30	1.99
	G_{17}	102	15	2.31

Table 2. Pion induced reactions fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02.

$\pi^- p \rightarrow \eta n$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[15]	$d\sigma/d\Omega$	70	20	1.47
[16]	$d\sigma/d\Omega$	84	30	2.98
$\pi^- p \rightarrow K^0 \Lambda$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[17]	$d\sigma/d\Omega$	300	30	0.90
[18, 19]	$d\sigma/d\Omega$	298	30	2.30
[18, 19]	P	355	30	1.77
[20]	β	72	70	1.06
$\pi^+ p \rightarrow K^+ \Sigma^+$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[21, 22, 23, 24, 25]	$d\sigma/d\Omega$	728	35	1.46
[21, 22, 23, 24, 25, 26]	P	351	30	1.57
[27]	β	7	600	2.04
$\pi^- p \rightarrow K^0 \Sigma^0$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[28]	$d\sigma/d\Omega$	259	30	0.98
[28]	P	95	30	1.30

Table 3. Reactions leading to 3-body final states included in the event-based likelihood fits; likelihood values for the solution BG2011-02. CB stands for CB-ELSA; CBT for CBELSA/TAPS.

$d\sigma/d\Omega(\pi^- p \rightarrow \pi^0 \pi^0 n)$	N_{data}	w_i	$-\ln L$
T=373 MeV	5248	10	-924
T=472 MeV	Crystal	10641	5
T=551 MeV	Ball [29]	41172	2.5
T=655 MeV	(BNL)	63514	2
T=691 MeV		30030	3.5
T=748 MeV		30379	4
$d\sigma/d\Omega(\gamma p \rightarrow \pi^0 \pi^0 p)$ CB [30, 31]	110601	4	-26953
$d\sigma/d\Omega(\gamma p \rightarrow \pi^0 \pi^0 p)$ CB [32]	10000	7	-5276
$d\sigma/d\Omega(\gamma p \rightarrow \pi^0 \eta p)$ CB [10, 33, 34]	17468	8	-5701
N_{data}	w_i	χ^2/N_{data}	
$\Sigma(\gamma p \rightarrow \pi^0 \pi^0 p)$ GRAAL [35]	128	35	1.11
$\Sigma(\gamma p \rightarrow \pi^0 \eta p)$ CBT [36]	180	15	2.40
$E(\gamma p \rightarrow \pi^0 \pi^0 p)$ GDH/A2 [37]	16	35	1.26
$I_c, I_s(\gamma p \rightarrow \pi^0 \pi^0 p)$ CBT [38]	210	10	1.45
$I_c, I_s(\gamma p \rightarrow \pi^0 \pi^0 p)$ CBT [39]	1000	10	1.71
N_{data}	w_i	χ^2/N_{data}	

Table 4. Observables from η photoproduction fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02.

$\gamma p \rightarrow \eta p$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[114] Crystal Ball @ MAMI	$d\sigma/d\Omega$	2400	2	1.30
[115] CBT	$d\sigma/d\Omega$	680	40	1.39
[116] CB	$d\sigma/d\Omega$	631	20	1.74
[117] GRAAL	Σ	51	10	1.81
[118] GRAAL	Σ	150	15	1.19
[119] CBT	Σ	34	20	0.82

Table 5. Observables from π photoproduction fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02.

$\gamma p \rightarrow \pi^0 p$	Observ.	N_{data}
[40] (TAPS@MAMI)	$d\sigma/d\Omega$	1692
[41, 42] (GDH A2)	$d\sigma/d\Omega$	164
[43] (GRAAL)	$d\sigma/d\Omega$	861
[44, 45] (CB)	$d\sigma/d\Omega$	1106
[46] (CLAS)	$d\sigma/d\Omega$	592
[47] (CBT)	$d\sigma/d\Omega$	540
[43, 48, 49, 50, 51, 52, 53, 54, 55]	Σ	1492
[56] (CBT)	Σ	374
[49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66]	T	389
[49, 50, 51, 66, 67, 68, 69, 70]	P	607
[71, 72]	G	75
[71]	H	71
[41, 42]	E	140
[69, 73]	$O_{x'}$	7
[69, 73]	$O_{z'}$	7
$\gamma p \rightarrow \pi^+ n$	Observ.	N_{data}
[74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85]	$d\sigma/d\Omega$	1583
[42, 86] (GDH A2)	$d\sigma/d\Omega$	408
[87] (CLAS)	$d\sigma/d\Omega$	484
[55, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98]	Σ	899
[93, 94, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109]	T	661
[93, 94, 110]	P	252
[72, 111, 112]	G	86
[111, 112, 113]	H	128
[42, 86]	E	231

Table 6. Hyperon photoproduction observables fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02.

$\gamma p \rightarrow K^+ \Lambda$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[120] CLAS	$d\sigma/d\Omega$	1320	16	0.69
[121] LEPS	Σ	45	10	2.11
[122] GRAAL	Σ	66	8	2.95
[120] CLAS	P	1270	8	1.82
[122] GRAAL	P	66	10	0.59
[123] GRAAL	T	66	15	1.62
[124] CLAS	C_x	160	15	1.52
[124] CLAS	C_z	160	15	1.58
[123] GRAAL	$O_{x'}$	66	12	1.95
[123] GRAAL	$O_{z'}$	66	15	1.66
$\gamma p \rightarrow K^+ \Sigma$	Observ.	N_{data}	w_i	χ_i^2/N_{data}
[125] CLAS	$d\sigma/d\Omega$	1590	3	1.44
[121] LEPS	Σ	45	10	1.23
[122] GRAAL	Σ	42	10	1.99
[125] CLAS	P	344	12	2.69
[124] CLAS	C_x	94	15	1.95
[124] CLAS	C_z	94	15	1.66
$\gamma p \rightarrow K^0 \Sigma^+$	Obsv.	N_{data}	w_i	χ_i^2/N_{data}
[126] CLAS	$d\sigma/d\Omega$	48	3	3.84
[127] SAPHIR	$d\sigma/d\Omega$	160	5	1.91
[128] CBT	$d\sigma/d\Omega$	72	10	0.76
[129] CBT	$d\sigma/d\Omega$	72	40	0.62
[128] CBT	P	72	15	0.90
[129] CBT	P	24	30	0.94
[129] CBT	Σ	15	50	1.73

tiparticle final states. Resonances with sizable coupling constants to πN and γN are thus unlikely to escape the fits even though further single and double polarization experiments are certainly needed to unambiguously constrain the contributing amplitudes. A few data sets were omitted for reasons discussed in [7]. The Tables list the reaction, the observables and references to the data, the number of data points, the weight with which the data are used in the fits, and the χ^2 per data point of our final solution BG2011-02. Multibody final states are fitted in an event-based likelihood fit. For these reactions, the log likelihood is given (see Eq. 17). The analysis was constrained by the total cross sections for $\pi^- p \rightarrow n\pi^+\pi^-$ and $\pi^+ p \rightarrow p\pi^0\pi^0$ from [130].

3 Partial wave analysis and definitions

The partial wave analysis method used in this analysis is described in detail in [131, 132]. A shorter survey can be found in [7]. In the Tables below we give pole parameters as well as Breit-Wigner parameters. Here, we give the precise definitions used to calculate the quantities given in the Tables.

The transition amplitude for a pion- or photo-produced reaction from the initial state $a = \pi N$ or γN and with b , e.g. AK^+ , as final state can be defined as

$$A_{ab} = K_{ac}(I - i\rho K)_{cb}^{-1} \quad (1)$$

where K is called K matrix and ρ is the phase space. A single resonance is described by a term

$$K_{ab} = \frac{g_a g_b}{M^2 - s}. \quad (2)$$

with g_a, g_b being coupling constants. In this case the equation (1) corresponds to the relativistic Breit-Wigner amplitude

$$A_{ab} = \frac{g_a g_b}{M^2 - s - i \sum_j g_j^2 \rho_j(s)} \quad (3)$$

where $M = M_{BW}$ is called Breit-Wigner mass. For $\sum_j g_j^2 \rho_j(s)$ replaced by $M\Gamma$, we obtain the non-relativistic Breit-Wigner amplitude. The pole position is defined as zero of the amplitude denominator in the complex plane

$$M^2 - s - i \sum_j g_j^2 \rho_j(s) = 0 \quad (4)$$

and the partial width Γ_a at $s = M^2$ (at the BW mass) is defined as

$$M\Gamma_a = g_a^2 \rho_a(M^2) \quad (5)$$

The helicity-dependent amplitude for photoproduction of the final state b can be written as

$$a_b^h(s) = \frac{A_{BW}^h g_b}{M^2 - s - i \sum_j g_j^2 \rho_j(s)}, \quad (6)$$

where A_{BW}^h are photo-production couplings e.g. helicity couplings in the helicity basis.

In general, the amplitude contains not only one resonance. In case of several resonances (index α), the K matrix can be written as:

$$K_{ab} = \sum_{\alpha} \frac{g_{\alpha}^{\alpha} g_b^{\alpha}}{M_{\alpha}^2 - s} + f_{ab}. \quad (7)$$

Here the background terms f_{ab} are added: they may be arbitrary functions of s and describe non-resonant transitions from the initial to the final state.

The position of the pole ($M_{pole} - i\frac{1}{2}\Gamma_{pole}$) can be found by calculation of the zeros of the denominator of a K-matrix amplitude in the complex s-plane [133]

$$\det(I - i\rho K) \prod_{\alpha} (M_{\alpha}^2 - s) = 0. \quad (8)$$

We define the residues for the transition amplitude by the contour integral of the amplitude around the pole position in the energy (\sqrt{s}) plane to

$$\begin{aligned} Res(a \rightarrow b) &= \int \frac{d\sqrt{s}}{2\pi i} \sqrt{\rho_a} A_{ab}(s) \sqrt{\rho_b} \\ &= \frac{1}{2M_p} \sqrt{\rho_a(M_p^2)} g_a^r g_b^r \sqrt{\rho_b(M_p^2)}. \end{aligned} \quad (9)$$

Here M_p is the position of the pole (complex number) and g_a^r are pole couplings. The elastic pole residue is defined as

$$Res(\pi N \rightarrow N\pi) = \frac{1}{2M_p} (g_{N\pi}^r)^2 \rho_{N\pi}(M_p^2) \quad (10)$$

In the pole position one has a full factorization of the amplitude:

$$Res^2(a \rightarrow b) = Res(a \rightarrow a) \times Res(b \rightarrow b) \quad (11)$$

The helicity-dependent amplitude for photoproduction of the final state b is calculated in the framework of P-vector approach:

$$a_b^h = P_a^h (I - i\rho K)_{jb}^{-1} \quad (12)$$

where:

$$P_a^h = \sum_{\alpha} \frac{A_{\alpha}^h g_a^{\alpha}}{M_{\alpha}^2 - s} + F_a. \quad (13)$$

and A_{α}^h is photo-coupling of the K-matrix pole α and F_a is a non-resonant transition. In the resonance pole the photo-couplings are defined as:

$$A^h g_b^r = \int \frac{ds}{2\pi i} a_b^h(s). \quad (14)$$

The helicity amplitudes $A^{1/2}, A^{3/2}$ (photo-couplings in the helicity basis), the coupling elastic residues, and the residues of the transition amplitudes are complex numbers. They become real and coincide with the conventional helicity amplitudes $A^{1/2}, A^{3/2}$, to half the elastic width $\Gamma_{N\pi}/2$, and to the

channel coupling $\frac{1}{2}\sqrt{T_i T_f}$ if a Breit-Wigner amplitude with constant width is used.

The elastic residue, which is proportional to $(g_{N\pi}^r)^2 \rho_{N\pi}(M_p^2)$, defines $g_{N\pi}^r$ up to a sign. This may lead to ambiguities if the phase is not properly defined: assume the phase of elastic residue would be $(180 \pm \epsilon)^\circ$ in two analyses. Due to eq. (14), the phase of the helicity amplitude depends on this definition. Since the phases of the elastic pole residue of most resonances are negative, we define in the case of elastic residues with a negative real part the phase of $g_{N\pi}^r$ clockwise.

In this article we also give some quantities which are related to properties of a relativistic Breit-Wigner amplitude. We define the Breit-Wigner amplitude by

$$A_{ab} = \frac{f^2 g_a^r g_b^r}{M_{BW}^2 - s - i f^2 \sum_a |g_a^r|^2 \rho_a(s)} \quad (15)$$

where M_{BW} and scaling factor f are calculated to reproduce exactly the pole position of the resonance. For a true Breit-Wigner amplitude, $f = 1$, and the definition in eq. (15) coincides with the one in eq. (3). In the case of a very fast growing phase volume, the Breit-Wigner mass and width can shift from the pole position by a large amount. For example, the Breit-Wigner mass of the Roper resonance is 60-80 MeV higher than the pole position and its Breit-Wigner width exceeds the pole width by about 150 MeV. In the 1600-1700 MeV region, the large phase volume leads to a very large Breit-Wigner widths and an appreciable shift in mass from the pole position (see for example [30]) if the ρN , $\Delta \pi$ (with large L), and $D_{15}(1520)\pi$ decay modes are taken into account explicitly. The visible width, e.g. in the $N\pi$ invariant mass spectrum, remains similar to the Breit-Wigner width. Clearly, the large phase volume effects are highly model dependent and possibly, they are artifacts of the formalism. We therefore decided to extract the Breit-Wigner parameters of resonances above the Roper resonance by approximating the phase volumes for the three body channels in eq.(15) as πN phase volume for the respective partial wave. This procedure conserves the branching ratio between three particle and πN channels at the resonance position and at the Breit-Wigner mass.

The Breit-Wigner helicity amplitude is defined as:

$$a_a^h = \frac{A_{BW}^h f g_a^r}{M_{BW}^2 - s - i f^2 \sum_a |g_a^r|^2 \rho_a(s)}, \quad (16)$$

where A_{BW}^h is calculated to reproduce the pion photo-production residues in the pole. In general this quantum is a complex number. However, for majority of resonances its phase deviates only little from 0 or 180 degrees.

4 Properties of baryon resonances

On the subsequent pages we present properties of nucleon and Δ resonances determined in this work. We give pole parameters: pole position (eq. 8), the complex helicity amplitudes $A^{1/2}$ and $A^{3/2}$ (eq. 14), the elastic pole residue (eq. 10) and residues for hadronic transition amplitudes (eq. 9).

The Tables also give properties of a relativistic Breit-Wigner amplitude (eq. 15), its helicity amplitudes (eq. 16), partial decay widths (eq. 5), and branching ratios for the decay into channel a by $\Gamma_a/\bar{\Gamma}$.

A large number of resonances is required to achieve a good description of all data sets. These resonances couple to a variety of different decay modes. The optimum set of parameters is determined in fits to the data of Tables 1-6. The fit minimizes the total log likelihood defined by

$$-\ln \mathcal{L}_{\text{tot}} = \left(\frac{1}{2} \sum w_i \chi_i^2 - \sum w_i \ln \mathcal{L}_i \right) \frac{\sum N_i}{\sum w_i N_i} \quad (17)$$

where the summation over binned data contributes to the χ^2 while unbinned data contribute to the likelihoods \mathcal{L}_i . Data with $p\pi^0\pi^0$ and $p\pi^0\eta$ in the final state - except those taken with polarized photons - are fitted event by event in order to take into account all possible correlations between the variables. For convenience of the reader, we quote differences in fit quality as χ^2 difference, with $\Delta\chi^2 = -2\Delta\mathcal{L}_{\text{tot}}$. For new data, the weight is increased from $w_i = 1$ until a visually acceptable fit is reached. Without weights, low-statistics data e.g. on polarization variables may be reproduced unsatisfactorily without significant deterioration of the total \mathcal{L}_{tot} . The likelihood function is normalized to avoid an artificial increase in statistics by the weighting factors.

Due to the incomplete data base with few double polarization observables only, the solution of the partial wave analysis is not unequivocal. Depending on the number of poles in the different partial waves and depending on start values of the fit, different minima of similar χ^2 are reached. However, most parameters are stable, only a few parameters undergo substantial changes. The solutions which have converged to minima of similar depth are stored; from the distribution of the fit results, typically more than ten, the mean value and the error is deduced. As error we assume the half-width of the distribution. In some cases, solutions exist with a distinct minimum forming a new class of results, and leading to a new set of parameters. Often, they cluster into two main solutions, called BG2011-01 and BG2011-02. The most significant difference can be found in the $1/2(3/2^+)$ wave where BG2011-02 finds two close-by resonances: $N(1900)3/2^+$, present in both types of solutions with slightly different parameters, and $N(1975)3/2^+$, present only in BG2011-02. Here, we give the properties of $N(1900)3/2^+$ only. Sizable differences between the BG2011-01 and BG2011-02 solutions are also observed in the $3/2^-$ (in particular for $N(1700)3/2^-$), $5/2^+$ and $7/2^+$ wave. The different solutions are discussed explicitly in [9]. Here, we give errors which cover both solutions. The two solutions give similar properties for $N(1880)1/2^+$ except for its helicity amplitude. Here, we list both solutions in the Tables.

The 1700 MeV region is complicated due to the presence of two important thresholds, $N(1520)3/2^-\pi$ and ΣK . $N(1520)3/2^-\pi$ in S-wave gives $3/2^+$ quantum numbers; in

Table 7. (Next pages) Summary of results of the BnGa partial wave analysis. The first blocks give quantities related to the pole of the resonance, the second blocks give Breit-Wigner parameters.

$N(1440)\frac{1}{2}^+$	or	$N(1440)P_{11}$	$N(1520)\frac{3}{2}^-$	or	$N(1520)D_{13}$
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$N(1440)\frac{1}{2}^+$ pole parameters (MeV)			$N(1520)\frac{3}{2}^-$ pole parameters (MeV)		
M_{pole}	1370 ± 4	Γ_{pole}	190 ± 7	M_{pole}	1507 ± 3
Elastic pole residue	48 ± 3	Phase	$-(78 \pm 4)^\circ$	Elastic pole residue	36 ± 3
Residue $\pi N \rightarrow N\sigma$	20 ± 5	Phase	$-(135 \pm 7)^\circ$	Residue $\pi N \rightarrow \Delta\pi_{L=0}$	18 ± 4
Residue $\pi N \rightarrow \Delta\pi$	26 ± 3	Phase	$(40 \pm 5)^\circ$	Residue $\pi N \rightarrow \Delta\pi_{L=2}$	14 ± 3
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.044 ± 0.007	Phase	$(142 \pm 5)^\circ$	$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.021 ± 0.004
				$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.132 ± 0.009
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$N(1440)\frac{1}{2}^+$ Breit-Wigner parameters (MeV)			$N(1520)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)		
M_{BW}	1430 ± 8	Γ_{BW}	365 ± 35	M_{BW}	1517 ± 3
$\text{Br}(\pi N)$	$62 \pm 3\%$			$\text{Br}(\pi N)$	$62 \pm 3\%$
$\text{Br}(N\sigma)$	$17 \pm 7\%$	$\text{Br}(\Delta\pi)$	$21 \pm 8\%$	$\text{Br}(\Delta\pi_{L=0})$	$19 \pm 4\%$
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.061 ± 0.008			$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.022 ± 0.004
				$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.131 ± 0.010
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$N(1535)\frac{1}{2}^-$	or	$N(1535)S_{11}$	$N(1650)\frac{1}{2}^-$	or	$N(1650)S_{11}$
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$N(1535)\frac{1}{2}^-$ pole parameters (MeV)			$N(1650)\frac{1}{2}^-$ pole parameters (MeV)		
M_{pole}	1501 ± 4	Γ_{pole}	134 ± 11	M_{pole}	1647 ± 6
Elastic pole residue	31 ± 4	Phase	$-(29 \pm 5)^\circ$	Elastic pole residue	24 ± 3
Residue $\pi N \rightarrow N\eta$	29 ± 4	Phase	$-(76 \pm 5)^\circ$	Residue $\pi N \rightarrow N\eta$	15 ± 2
Residue $\pi N \rightarrow \Delta\pi$	8 ± 3	Phase	$(145 \pm 17)^\circ$	Residue $\pi N \rightarrow \Lambda K$	11 ± 3
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.116 ± 0.010	Phase	$(7 \pm 6)^\circ$	Residue $\pi N \rightarrow \Delta\pi$	12 ± 3
				$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.033 ± 0.007
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$N(1535)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)			$N(1650)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)		
M_{BW}	1519 ± 5	Γ_{BW}	128 ± 14	M_{BW}	1651 ± 6
$\text{Br}(\pi N)$	$54 \pm 5\%$			$\text{Br}(N\pi)$	$51 \pm 4\%$
$\text{Br}(N\eta)$	$33 \pm 5\%$	$\text{Br}(\Delta\pi)$	$2.5 \pm 1.5\%$	$\text{Br}(\Lambda K)$	$10 \pm 5\%$
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.105 ± 0.010			$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.033 ± 0.007
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$N(1675)\frac{5}{2}^-$	or	$N(1675)D_{15}$	$N(1680)\frac{5}{2}^+$	or	$N(1680)F_{15}$
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$N(1675)\frac{5}{2}^-$ pole parameters (MeV)			$N(1680)\frac{5}{2}^+$ pole parameters (MeV)		
M_{pole}	1654 ± 4	Γ_{pole}	151 ± 5	M_{pole}	1676 ± 6
Elastic pole residue	28 ± 1	Phase	$-(26 \pm 4)^\circ$	Elastic pole residue	43 ± 4
Residue $\pi N \rightarrow \Delta\pi$	25 ± 5	Phase	$(82 \pm 10)^\circ$	Residue $\pi N \rightarrow \Delta\pi_{L=1}$	8 ± 3
Residue $\pi N \rightarrow N\sigma$	11 ± 4	Phase	$(132 \pm 18)^\circ$	Residue $\pi N \rightarrow N\sigma$	14 ± 3
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.024 ± 0.003	Phase	$-(16 \pm 5)^\circ$	$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.013 ± 0.004
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.026 ± 0.008	Phase	$-(19 \pm 6)^\circ$	$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.134 ± 0.005
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$N(1675)\frac{5}{2}^-$ Breit-Wigner parameters (MeV)			$N(1680)\frac{5}{2}^+$ Breit-Wigner parameters (MeV)		
M_{BW}	1664 ± 5	Γ_{BW}	152 ± 7	M_{BW}	1689 ± 6
$\text{Br}(N\pi)$	$40 \pm 3\%$			$\text{Br}(N\pi)$	$64 \pm 5\%$
$\text{Br}(\Delta\pi)$	$33 \pm 8\%$	$\text{Br}(N\sigma)$	$7 \pm 3\%$	$\text{Br}(\Delta\pi_{L=1})$	$10 \pm 3\%$
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.024 ± 0.003	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.025 ± 0.007	$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.013 ± 0.003
				$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.135 ± 0.006

$N(1700)\frac{3}{2}^-$	or	$N(1700)D_{13}$	$N(1710)\frac{1}{2}^+$	or	$N(1710)P_{11}$
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$N(1700)\frac{3}{2}^-$ pole parameters (MeV)					
M_{pole}	1770±40	Γ_{pole}	420±180		
Elastic pole residue	50±40	Phase	-(100±40)°		
Residue $\pi N \rightarrow \Delta \pi_{L=0}$	75±50	Phase	-(60±40)°		
Residue $\pi N \rightarrow \Delta \pi_{L=2}$	18±12	Phase	(90±35)°		
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.044±0.020	Phase	(85±45)°		
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	-0.037±0.012	Phase	(0±30)°		
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$N(1700)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)					
M_{BW}	1790±40	Γ_{BW}	390±140		
$\text{Br}(\pi N)$	12±5%				
$\text{Br}(\Delta \pi_{L=0})$	72±16%	$\text{Br}(\Delta \pi_{L=2})$	5±4%		
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.041±0.017	$A_{\text{BW}}^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	-0.034±0.013		
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$N(1720)\frac{3}{2}^+$	or	$N(1720)P_{13}$	$N(1860)\frac{5}{2}^+$	or	$N(1860)F_{15}$
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$N(1720)\frac{3}{2}^+$ pole parameters (MeV)					
M_{pole}	1660±30	Γ_{pole}	450±100		
Elastic pole residue	22±8	Phase	-(115±30)°		
Residue $\pi N \rightarrow N\eta$	7±5	Phase	not defined		
Residue $\pi N \rightarrow \Lambda K$	14±10	Phase	-(150±45)°		
Residue $\pi N \rightarrow \Delta \pi_{L=1}$	64±25	Phase	(80±40)°		
Residue $\pi N \rightarrow \Delta \pi_{L=3}$	8±8	Phase	not defined		
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.110±0.045	Phase	(0±40)°		
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.150±0.035	Phase	(65±35)°		
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$N(1720)\frac{3}{2}^+$ Breit-Wigner parameters (MeV)					
M_{BW}	1690 ⁺⁷⁰ ₋₃₅	Γ_{BW}	420±100		
$\text{Br}(N\pi)$	10±5%	$\text{Br}(N\eta)$	3±2%		
$\text{Br}(\Delta \pi_{L=1})$	75±15%	$\text{Br}(\Delta \pi_{L=3})$	2±2%		
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.110±0.045	$A_{\text{BW}}^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.150±0.030		
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$N(1875)\frac{3}{2}^-$	or	$N(1875)D_{13}$	$N(1880)\frac{1}{2}^+$	or	$N(1880)P_{11}$
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$N(1875)\frac{3}{2}^-$ pole parameters (MeV)					
M_{pole}	1860±25	Γ_{pole}	200±20		
Elastic pole residue	2.5±1.0	Phase	not defined		
Residue $\pi N \rightarrow \Lambda K$	1.5±1.0	Phase	not defined		
Residue $\pi N \rightarrow \Sigma K$	5±3	Phase	not defined		
Residue $\pi N \rightarrow N\sigma$	8±3	Phase	-(170±65)°		
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.018±0.008	Phase	-(100±60)°		
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.010±0.004	Phase	(180±30)°		
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$N(1875)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)					
M_{BW}	1880±20	Γ_{BW}	200±25		
$\text{Br}(N\pi)$	3±2%	$\text{Br}(N\eta)$	5±2%		
$\text{Br}(\Lambda K)$	4±2%	$\text{Br}(\Sigma K)$	15±8%		
$\text{Br}(N\sigma)$	60±12%				
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.018±0.010	$A_{\text{BW}}^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	-0.009±0.005		
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$N(1880)\frac{1}{2}^+$ pole parameters (MeV)					
M_{pole}	1860±35	Γ_{pole}	250±70		
Elastic pole residue	6±4	Phase	(80±65)°		
Residue $\pi N \rightarrow \eta N$	13±8	Phase	-(75±55)°		
Residue $\pi N \rightarrow \Lambda K$	4±3	Phase	(40±40)°		
Residue $\pi N \rightarrow \Sigma K$	13±7	Phase	(95±40)°		
Residue $\pi N \rightarrow \Delta \pi$	27±13	Phase	-(150±50)°		
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.014±0.003 ⁽⁰¹⁾	Phase	-(130±60)°		
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.036±0.012 ⁽⁰²⁾	Phase	(15±20)°		
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$N(1880)\frac{1}{2}^+$ Breit-Wigner parameters (MeV)					
M_{BW}	1870±35	Γ_{BW}	235±65		
$\text{Br}(\pi N)$	5±3%	$\text{Br}(\eta N)$	25 ⁺³⁰ ₋₂₀ %		
$\text{Br}(\Lambda K)$	2±1%	$\text{Br}(\Sigma K)$	17±7%		
$\text{Br}(\Delta \pi)$	29±12%				
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)			-0.013±0.003 ⁽⁰¹⁾		
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)			0.034±0.011 ⁽⁰²⁾		

$N(1895)\frac{1}{2}^-$	or	$N(1895)S_{11}$	$N(1900)\frac{3}{2}^+$	or	$N(1900)P_{13}$
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$N(1895)\frac{1}{2}^-$ pole parameters (MeV)					
M_{pole}	1900 ± 15	Γ_{pole}	90^{+30}_{-15}		
Elastic pole residue	1 ± 1	Phase	not defined		
Residue $\pi N \rightarrow \eta N$	3 ± 2	Phase	$(40 \pm 20)^\circ$		
Residue $\pi N \rightarrow K\Lambda$	2 ± 1	Phase	$-(90 \pm 30)^\circ$		
Residue $\pi N \rightarrow K\Sigma$	3 ± 2	Phase	$(40 \pm 30)^\circ$		
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.012 ± 0.006	Phase	$(120 \pm 50)^\circ$		
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$N(1895)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)					
M_{BW}	1895 ± 15	Γ_{BW}	90^{+30}_{-15}		
$\text{Br}(\pi N)$	$2 \pm 1\%$	$\text{Br}(\eta N)$	$21 \pm 6\%$		
$\text{Br}(K\Lambda)$	$18 \pm 5\%$	$\text{Br}(K\Sigma)$	$13 \pm 7\%$		
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.011 ± 0.006				
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$N(1990)\frac{7}{2}^+$	or	$N(1990)F_{17}$	$N(2000)\frac{5}{2}^+$	or	$N(2000)F_{15}$
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$N(1990)\frac{7}{2}^+$ pole parameters (MeV)					
M_{pole}	2030 ± 65	Γ_{pole}	240 ± 60		
Elastic pole residue	2 ± 1	Phase	$(125 \pm 65)^\circ$		
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.042 ± 0.014	Phase	$-(30 \pm 20)^\circ$		
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.058 ± 0.012	Phase	$-(35 \pm 25)^\circ$		
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$N(1990)\frac{7}{2}^+$ Breit-Wigner parameters (MeV)					
M_{BW}	2060 ± 65	Γ_{BW}	240 ± 50		
$\text{Br}(\pi N)$	$2 \pm 1\%$				
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.040 ± 0.012	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.057 ± 0.012		
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$N(2060)\frac{5}{2}^-$	or	$N(2060)D_{15}$	$N(2150)\frac{3}{2}^-$	or	$N(2150)D_{13}$
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$N(2060)\frac{5}{2}^-$ pole parameters (MeV)					
M_{pole}	2040 ± 15	Γ_{pole}	390 ± 25		
Elastic pole residue	19 ± 5	Phase	$-(125 \pm 20)^\circ$		
Residue $\pi N \rightarrow \eta N$	12 ± 6	Phase	$(40 \pm 25)^\circ$		
Residue $\pi N \rightarrow K\Lambda$	1 ± 0.5	Phase	not defined		
Residue $\pi N \rightarrow K\Sigma$	7 ± 4	Phase	$-(70 \pm 30)^\circ$		
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.065 ± 0.012	Phase	$(15 \pm 8)^\circ$		
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.055^{+15}_{-35}	Phase	$(15 \pm 10)^\circ$		
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$N(2060)\frac{5}{2}^-$ Breit-Wigner parameters (MeV)					
M_{BW}	2060 ± 15	Γ_{BW}	375 ± 25		
$\text{Br}(\pi N)$	$8 \pm 2\%$	$\text{Br}(\eta N)$	$4 \pm 2\%$		
$\text{Br}(K\Sigma)$	$3 \pm 2\%$				
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.067 ± 0.015	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.055 ± 0.020		
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$N(2150)\frac{3}{2}^-$ pole parameters (MeV)					
M_{pole}	2110 ± 50	Γ_{pole}	340 ± 45		
Elastic pole residue	13 ± 3	Phase	$-(20 \pm 10)^\circ$		
Residue $\pi N \rightarrow K\Lambda$	5 ± 2	Phase	$(100 \pm 30)^\circ$		
Residue $\pi N \rightarrow K\Sigma$	3 ± 2	Phase	$-(50 \pm 40)^\circ$		
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.125 ± 0.045	Phase	$-(55 \pm 20)^\circ$		
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.150 ± 0.060	Phase	$-(35 \pm 15)^\circ$		
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$N(2150)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)					
M_{BW}	2150 ± 60	Γ_{BW}	330 ± 45		
$\text{Br}(\pi N)$	$6 \pm 2\%$				
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.130 ± 0.045	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.150 ± 0.055		
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$N(2190)\frac{7}{2}^-$	or	$N(2190)G_{17}$	$N(2220)\frac{9}{2}^+$	or	$N(2220)H_{19}$			
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$N(2190)\frac{7}{2}^-$ pole parameters (MeV)								
M_{pole}	2150 ± 25	Γ_{pole}	330 ± 30					
Elastic pole residue	30 ± 5	Phase	$(30 \pm 10)^\circ$					
Residue $\pi N \rightarrow K\Lambda$	4.9 ± 1.5	Phase	$(20 \pm 15)^\circ$					
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.063 ± 0.007	Phase	$-(170 \pm 15)^\circ$					
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.035 ± 0.020	Phase	$(25 \pm 10)^\circ$					
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$N(2190)\frac{7}{2}^-$ Breit-Wigner parameters (MeV)								
M_{BW}	2180 ± 20	Γ_{BW}	335 ± 40					
$\text{Br}(\pi N)$	$16 \pm 2\%$	$\text{Br}(K\Lambda)$	$0.5 \pm 0.3\%$					
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.065 ± 0.008	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.035 ± 0.017					
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$N(2250)\frac{9}{2}^-$	or	$N(2250)G_{19}$	$N(2220)\frac{9}{2}^+$ Breit-Wigner parameters (MeV)					
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$N(2250)\frac{9}{2}^-$ pole parameters (MeV)								
M_{pole}	2195 ± 45	Γ_{pole}	470 ± 50					
Elastic pole residue	26 ± 5	Phase	$-(38 \pm 25)^\circ$					
$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	< 0.010	Phase	not defined					
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	< 0.010	Phase	not defined					
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$N(2250)\frac{9}{2}^-$ Breit-Wigner parameters (MeV)								
M_{BW}	2280 ± 40	Γ_{BW}	520 ± 50					
$\text{Br}(\pi N)$	$12 \pm 4\%$							
$ A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}}) < 0.010$		$ A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}}) < 0.010$						
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$\Delta(1232)\frac{3}{2}^+$	or	$\Delta(1232)P_{33}$	$\Delta(1600)\frac{3}{2}^+$	or	$\Delta(1600)P_{33}$			
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$\Delta(1232)\frac{3}{2}^+$ pole parameters (MeV)								
M_{pole}	1210.5 ± 1.0	Γ_{pole}	99 ± 2					
Elastic pole residue	51.6 ± 0.6	Phase	$-(46 \pm 1)^\circ$					
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$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.131 ± 0.0035	Phase	$-(19 \pm 2)^\circ$					
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	-0.254 ± 0.0045	Phase	$-(9 \pm 1)^\circ$					
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$\Delta(1232)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)								
M_{BW}	1228 ± 2	Γ_{BW}	110 ± 3					
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$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.131 ± 0.004	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	-0.254 ± 0.005					
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$\Delta(1600)\frac{3}{2}^+$ pole parameters (MeV)								
M_{pole}	1498 ± 25	Γ_{pole}	230 ± 50					
Elastic pole residue	11 ± 6	Phase	$-(160 \pm 33)^\circ$					
Residue $\pi N \rightarrow \Delta\pi_{L=1}$	18 ± 15	Phase	$(154 \pm 40)^\circ$					
Residue $\pi N \rightarrow \Delta\pi_{L=3}$	1 ± 1	Phase						
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$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	0.053 ± 0.010	Phase	$(130 \pm 25)^\circ$					
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	0.041 ± 0.011	Phase	$(165 \pm 17)^\circ$					
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$\Delta(1600)\frac{3}{2}^+$ Breit-Wigner parameters (MeV)								
M_{BW}	1510 ± 20	Γ_{BW}	220 ± 45					
$\text{Br}(N\pi)$	$12 \pm 5\%$							
$\text{Br}(\Delta\pi_{L=1})$	$78 \pm 6\%$	$\text{Br}(\Delta\pi_{L=3})$	$2 \pm 2\%$					
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.050 ± 0.009	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	-0.040 ± 0.012					

$\boxed{\Delta(1620)\frac{1}{2}^-}$	or	$\boxed{\Delta(1700)\frac{3}{2}^-}$	or	$\Delta(1700)D_{33}$
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$\Delta(1620)\frac{1}{2}^-$ pole parameters (MeV)				
M_{pole}	1597 ± 4	Γ_{pole}	130 ± 9	
Elastic pole residue	18 ± 2	Phase	$-(100 \pm 5)^\circ$	
Residue $\pi N \rightarrow \Delta\pi$	25 ± 5	Phase	$-(85 \pm 30)^\circ$	
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.052 ± 0.005	Phase	$-(9 \pm 9)^\circ$	
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$\Delta(1620)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)				
M_{BW}	1600 ± 8	Γ_{BW}	130 ± 11	
$\text{Br}(N\pi)$	$28 \pm 3\%$	$\text{Br}(\Delta\pi)$	$60 \pm 12\%$	
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.052 ± 0.005			
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$\boxed{\Delta(1900)\frac{1}{2}^+}$	or	$\boxed{\Delta(1905)\frac{5}{2}^+}$	or	$\Delta(1905)F_{35}$
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$\Delta(1900)\frac{1}{2}^+$ pole parameters (MeV)				
M_{pole}	1845 ± 25	Γ_{pole}	300 ± 45	
Elastic pole residue	10 ± 3	Phase	$-(125 \pm 20)^\circ$	
Residue $\pi N \rightarrow \Sigma K$	10 ± 3	Phase	$-(50 \pm 30)^\circ$	
Residue $\pi N \rightarrow \Delta\pi$	15 ± 10	Phase	$(110 \pm 20)^\circ$	
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.059 ± 0.016	Phase	$(60 \pm 25)^\circ$	
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$\Delta(1900)\frac{1}{2}^+$ Breit-Wigner parameters (MeV)				
M_{BW}	1840 ± 30	Γ_{BW}	300 ± 45	
$\text{Br}(N\pi)$	$7 \pm 3\%$	$\text{Br}(\Sigma K)$	$5 \pm 3\%$	
$\text{Br}(\Delta\pi)$	$58 \pm 25\%$			
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.059 ± 0.016			
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$\boxed{\Delta(1910)\frac{1}{2}^+}$	or	$\boxed{\Delta(1920)\frac{3}{2}^+}$	or	$\Delta(1920)P_{33}$
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$\Delta(1910)\frac{1}{2}^+$ pole parameters (MeV)				
M_{pole}	1850 ± 40	Γ_{pole}	350 ± 45	
Elastic pole residue	24 ± 6	Phase	$-(145 \pm 30)^\circ$	
Residue $\pi N \rightarrow \Sigma K$	12 ± 4	Phase	$-(110 \pm 30)^\circ$	
Residue $\pi N \rightarrow \Delta\pi$	30 ± 14	Phase	$(95 \pm 40)^\circ$	
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.023 ± 0.009	Phase	$(40 \pm 90)^\circ$	
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$\Delta(1910)\frac{1}{2}^+$ Breit-Wigner parameters (MeV)				
M_{BW}	1860 ± 40	Γ_{BW}	350 ± 55	
$\text{Br}(N\pi)$	$12 \pm 3\%$	$\text{Br}(\Sigma K)$	$9 \pm 5\%$	
$\text{Br}(\Delta\pi)$	$60 \pm 20\%$			
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.022 ± 0.009			
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$\Delta(1920)\frac{3}{2}^+$ pole parameters (MeV)				
M_{pole}	1890 ± 30	Γ_{pole}	300 ± 60	
Elastic pole residue	17 ± 8	Phase	$-(40 \pm 20)^\circ$	
Residue $\pi N \rightarrow \Sigma K$	14 ± 7	Phase	$(80 \pm 40)^\circ$	
Residue $\pi N \rightarrow \Delta\eta$	27 ± 12	Phase	$(70 \pm 20)^\circ$	
Residue $\pi N \rightarrow \Delta\pi_{L=1}$	30 ± 13	Phase	$-(120 \pm 30)^\circ$	
Residue $\pi N \rightarrow \Delta\pi_{L=3}$	44 ± 14	Phase	$-(95 \pm 35)^\circ$	
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	$0.130_{-0.060}^{+0.030}$	Phase	$-(65 \pm 20)^\circ$	
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	$0.115_{-0.050}^{+0.025}$	Phase	$-(160 \pm 20)^\circ$	
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$\Delta(1920)\frac{3}{2}^+$ Breit-Wigner parameters (MeV)				
M_{BW}	1900 ± 30	Γ_{BW}	310 ± 60	
$\text{Br}(N\pi)$	$8 \pm 4\%$	$\text{Br}(\Sigma K)$	$4 \pm 2\%$	
$\text{Br}(\Delta\eta)$	$15 \pm 8\%$			
$\text{Br}(\Delta\pi_{L=1})$	$22 \pm 9\%$	$\text{Br}(\Delta\pi_{L=3})$	$45 \pm 14\%$	
$A_{\text{BW}}^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	$0.130_{-0.060}^{+0.030}$			
$A_{\text{BW}}^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	$-0.115_{-0.050}^{+0.025}$			

$\Delta(1940)\frac{3}{2}^-$	or	$\Delta(1940)D_{33}$	$\Delta(1950)\frac{7}{2}^+$	or	$\Delta(1950)F_{37}$
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$\Delta(1940)\frac{3}{2}^-$ pole parameters (MeV)					
M_{pole}	1990^{+100}_{-50}	Γ_{pole}	450 ± 90		
Elastic pole residue	4 ± 4	Phase			
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$\Delta(1940)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)					
M_{BW}	1995^{+105}_{-60}	Γ_{BW}	450 ± 100		
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$\Delta(1950)\frac{7}{2}^+$ pole parameters (MeV)					
M_{pole}	1890 ± 4	Γ_{pole}	243 ± 8		
Elastic pole residue	58 ± 2	Phase	$-(24 \pm 3)^\circ$		
Residue $\pi N \rightarrow \Sigma K$	6 ± 1	Phase	$-(65 \pm 25)^\circ$		
Residue $\pi N \rightarrow \Delta \pi_{L=3}$	14 ± 4	Phase	$(12 \pm 10)^\circ$		
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$A^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.072 ± 0.004	Phase	$-(7 \pm 5)^\circ$		
$A^{3/2} (\text{GeV}^{-\frac{1}{2}})$	-0.096 ± 0.005	Phase	$-(7 \pm 5)^\circ$		
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$\Delta(1950)\frac{7}{2}^+$ Breit-Wigner parameters (MeV)					
M_{BW}	1915 ± 6	Γ_{BW}	246 ± 10		
$\text{Br}(N\pi)$	$45 \pm 2\%$	$\text{Br}(\Sigma K)$	$0.4 \pm 0.1\%$		
$\text{Br}(\Delta \pi_{L=3})$	$2.8 \pm 1.4\%$				
$A_{\text{BW}}^{1/2} (\text{GeV}^{-\frac{1}{2}})$	-0.071 ± 0.004	$A_{\text{BW}}^{3/2} (\text{GeV}^{-\frac{1}{2}})$	-0.094 ± 0.005		

deed, we find a strong $N(1720)3/2^+ \rightarrow N(1520)3/2^-\pi$ coupling. There seems to be a sizable $N(1720)3/2^+ \rightarrow \Lambda K$ coupling as well; the latter decay requires $L=1$. $N(1710)1/2^+$ may also have a significant ΛK coupling. A detailed study is required of the analytic structure of these two resonances in the threshold region. We have not included $\Delta(1750)1/2^+$ in the Tables below. We find no trace of evidence for this resonance and doubt that it exists. At present, the results on $\Delta(1940)3/2^-$ from $\gamma p \rightarrow p2\pi^0$ and $\gamma p \rightarrow p\pi^0\eta$ are not consistent. Also this issue needs further studies. At present, we give generous errors.

A few “new” resonances are reported. “New” does not mean, that resonances with these quantum numbers and similar masses and widths have not been reported before. But so far, these resonances have not been included in the Review of Particle Properties. These resonances are

$$N(1880)\frac{1}{2}^+, N(1860)\frac{5}{2}^+, N(1895)\frac{1}{2}^-, \\ N(1875)\frac{3}{2}^-, N(2150)\frac{3}{2}^-, \text{ and } N(2060)\frac{5}{2}^-.$$

Yet, $N(2150)3/2^-$ could be the 2^* resonance $N(2080)3/2^-$, and $N(2060)5/2^-$ could be related to $N(2200)5/2^-$, with 2^* as well, of the Particle Data Group.

The $N(1880)1/2^+$ resonance was first suggested when data on $\gamma p \rightarrow \Sigma^+ K_s^0$ from the CBELSA collaboration [128] were included in the BnGa partial wave analysis. $N(1975)3/2^+$ emerges from BNGA2011-02 only; it was first reported in [9]. Early evidence for $N(1860)5/2^+$ has been reported with Breit-Wigner parameters ($M_{\text{BW}}, \Gamma_{\text{BW}}$) equal to $(1882 \pm 10; 95 \pm 20)$ [2,5], $1903 \pm 87; 490 \pm 310$ [135], and $(1817.7; 117.6)$ [5]. Evidence for $N(1895)1/2^-$ has been reported by Höhler *et al.* [134] giving Breit-Wigner parameters of $M_{\text{BW}} = 1880 \pm 20, \Gamma_{\text{BW}} = 95 \pm 30$ MeV for a pole in the $I(J^P) = 1/2(1/2^-)$ wave. Manley *et al.* [135] found a broad state, $M_{\text{BW}} = 1928 \pm 59, \Gamma_{\text{BW}} = 414 \pm 157$ MeV. Vrana *et al.* [136] reported $M_{\text{BW}} = 1822 \pm 43, \Gamma_{\text{BW}} = 246 \pm 185$ MeV. A third and a forth pole in the $I(J^P) = 1/2(1/2^-)$ wave was suggested in [137]. The third pole was given with mass and width of $M_{\text{pole}} = 1733$ MeV; $\Gamma_{\text{pole}} = 180$ MeV, and in [138] with $M_{\text{pole}} = 1745 \pm 80, \Gamma_{\text{pole}} = 220 \pm 95$ MeV. The latter pole was also seen by Cutkosky

et al. [4] at $M_{\text{pole}} = 2150 \pm 70, \Gamma_{\text{pole}} = 350 \pm 100$ MeV and confirmed by Tiator *et al.* [137].

In the $\frac{1}{2}(\frac{3}{2}^-)$ wave, Cutkosky *et al.* [4] reported two resonances, the lower mass state at $M_{\text{BW}} = 1880 \pm 100, \Gamma_{\text{BW}} = 180 \pm 60$ MeV, the higher mass pole at $M_{\text{BW}} = 2060 \pm 60, \Gamma_{\text{BW}} = 300 \pm 10$ MeV. Saxon *et al.* [139] and Bell *et al.* [140] observed a $\frac{1}{2}(\frac{3}{2}^-)$ resonance in the reaction $\pi^- p \rightarrow \Lambda K^0$ at $(1900; 240)$ MeV and $(1920; 320)$ MeV, respectively. Based on SAPHIR data on $\gamma p \rightarrow \Lambda K^+$ [141], Mart and Bennhold claimed evidence for a $\frac{1}{2}(\frac{3}{2}^-)$ resonance at 1895 MeV [142] which was confirmed by us on a richer data base in [143,144], with mass and width of $(1875 \pm 25; 80 \pm 20)$ MeV, respectively. The high-mass $N_{3/2}^-$ was also seen in [143,144] with $(2166^{+25}_{-50}; \Gamma = 300 \pm 65)$ MeV and in [145] with $(2100 \pm 20; 200 \pm 50)$ MeV.

5 Significance and rating

In Table 8 we give our rating of the evidence with which baryon resonances are observed. By definition,

- **** Existence is certain, and properties are at least fairly well explored.
- *** Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions *etc.* are not well determined.
- ** Evidence of existence is only fair.
- * Evidence of existence is poor.

The significance of a resonance and of its decay modes is estimated from three sources: (i) from the increase in χ^2 when a resonance is removed from the fit, both the overall increase in χ^2 , and the increase in χ^2 in specific final states, (ii) from the stability of the fit result when the hypothesis (e.g. number of poles in a given partial wave) is changed, and (iii) from the errors in the definition of masses, widths, residues, photocouplings, *etc.* As a rule we give 1^* when a decay mode is seen with a significance of 2σ , 2^* for a significance of 3.5σ , and 3^* for a significance of 5σ . As there are ambiguous solutions, we do not assign 4^* for decays derived from photoproduction. In some cases, the errors are large, and the significance is high.

Table 8. Star rating suggested for baryon resonances and their decays. Ratings of the Particle Data Group are given as *; additional stars suggested from this analysis are represented by *; (*) stands for stars which should be removed.

	all	πN	γN	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\sigma$
$N(1440)\frac{1}{2}^+$	****	****	****	(*)			***	***
$N(1710)\frac{1}{2}^+$	***	***	***	***	***	***	**(*)	
$N(1880)\frac{1}{2}^+$	**	*	*		**	**		
$N(1535)\frac{1}{2}^-$	****	****	****	****			*	
$N(1650)\frac{1}{2}^-$	****	****	***	***	***	***	**	**(*)
$N(1895)\frac{1}{2}^-$	**	*	**	**	**	**	*	
$N(1720)\frac{3}{2}^+$	****	****	****	****	**	**	**	***
$N(1900)\frac{3}{2}^+$	***	**	****	**	****	**	**	
$N(1520)\frac{3}{2}^-$	****	****	****	***			****	
$N(1700)\frac{3}{2}^-$	***	**	**	*	*(*)	*	***	
$N(1875)\frac{3}{2}^-$	***	*	****		***	**		***
$N(2150)\frac{3}{2}^-$	**	**	**		**		**	
$N(1680)\frac{5}{2}^+$	****	****	****	*			**(*)	**
$N(1860)\frac{5}{2}^+$	*	*	*					
$N(2000)\frac{5}{2}^+$	***	*(*)	**	**	**	**	*	
$N(1675)\frac{5}{2}^-$	****	****	***(*)	*	*		***(*)	*
$N(2060)\frac{5}{2}^-$	***	**	****	*		**		
$N(1990)\frac{7}{2}^+$	**	*(*)	**					
$N(2190)\frac{7}{2}^-$	****	****	***		**			
$N(2220)\frac{9}{2}^+$	****	****						
$N(2250)\frac{9}{2}^-$	****	****						
$\Delta(1910)\frac{1}{2}^+$	****	****	**		**	**		
$\Delta(1620)\frac{1}{2}^-$	****	****	***				****	
$\Delta(1900)\frac{1}{2}^-$	**	**	**		**	**		
$\Delta(1232)\frac{3}{2}^+$	****	****	****					
$\Delta(1600)\frac{3}{2}^+$	***	***	***				***	
$\Delta(1920)\frac{3}{2}^+$	***	***	**		***	**		
$\Delta(1700)\frac{3}{2}^-$	***	***	***				**	
$\Delta(1940)\frac{3}{2}^-$	*	*	**				* from $\Delta\eta$	
$\Delta(1905)\frac{5}{2}^+$	****	****	****		***	**(*)		
$\Delta(1950)\frac{7}{2}^+$	****	****	***		***	***		

This happens, if there are two solutions which give different values for an observable, e.g. for its photoproduction amplitude. Without the resonance, the photoproduction data cannot be described; hence we are sure that the resonance is needed. But the actual value may be less certain. The star rating reflects our estimate how safe we are in claiming the existence of the resonance from photoproduction data; the error gives the range of values of resonance properties which might be assigned to a given resonance.

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